Set-01

Sixth Semester B.Sc. Degree Model Question Paper

(CBCS Scheme)

Mathematics

Paper 6.1 – Complex Analysis and Numerical Methods

Time: 3 Hours **Max. Marks : 90**

Instruction to Candidates: Answer All the questions.

Part-A

I. Answer any SIX of the following. (6 x 2 =12)

1. Show that amp$(z-1)=\frac{\pi}{2}$ Represent a line parallel to imaginary axis.
2. Show that $u=e^{x}siny$ is a harmonic function.
3. Define power series.
4. Evaluate $\int_0^{3+i} z^2 dz$ along the line $3y=x$ where $z=x+iy$.
5. State Cauchy's Integral theorem.
6. Prove that $E=e^{hd}$ where *h* is the interval of differences.
7. Evaluate $\int_0^1 e^{x} dx$ approximately in steps of 0.2 using Trapezoidal rule.

Part-B

II. Answer any SIX of the following . (6 x 3 = 18)

1. Show that $arg(\frac{\bar{z}}{z})=\frac{\pi}{2}$ represent a line through the origin .
2. Find the analytic function whose real part is x^3-3xy^2.
3. Define absolute convergence and radius of convergence of a power series.
4. Evaluate $\int_c (x^2-iy^2)dz$ along $y=2x^2$ from (1,2) to (2,8).
5. Evaluate $\int_C \frac{e^{az}}{z^2+1} dz$ where c is the circle $|z|=2$
6. Find the 7th term of the sequence 7,15,35,72,131,271 by constructing the difference table.
7. Using Newton Gregory Forward Interpolation Formula find $f(8.2)$ from the table

x	8.0	8.5	9.0	9.5	10.0
f(x)	50	57	64	71	78

PART-C

III Answer any FOUR of the following. **(4 x 5 = 20)**

1. Show that $arg(\frac{z-1}{z+2}) = \frac{\pi}{3}$ represents a circle. Find the center and its radius.
2. If (r, θ) are the polar co-ordinates of the function $f(z) = u(r, \theta) + iv(r, \theta)$ then show that $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$, $\frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r}$.
3. Show that $f(z) = coshz$ is analytic and $f'(z) = sinhz$.
4. Find the analytic function, $f(z) = u + iv$ given $u - v = (x - y)(x^2 + 4xy + y^2)$.
5. Find the radius of convergence of $\sum 2^{\sqrt{n}} z^n$.

IV. Answer any FOUR of the following. **(4 x 5 = 20)**

6. Evaluate $\int_C \bar{z}\, dz$ where C is given by two line segment joining $z = 0$ and $z = 2i$ and then $z = 2i$ to $z = 4 + 2i$.
7. State and prove Cauchy's integral formula.
8. Evaluate $\int_C \frac{z-1}{(z+1)(z-2)} dz$, where C is the circle $|z| = 3$.
9. Evaluate $\int_C \frac{dz}{(z^2+4)^2}$ where C is the circle $|z - i| = 2$.
10. State and prove Liouville's theorem.

V. Answer any FOUR of the following. **(4 x 5 = 20)**

11. Find the n^{th} difference of $sin(ax + b)$.
12. Using Lagranges interpolation formula find $f(0.5)$ from the table

x	0	1	2	5
f(x)	2	3	12	14.7

13. Using Newton divided difference formula find the value of $f(10)$ from the following table

x	1	2	4	7	12
f(x)	22	30	82	106	216

14. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 51$ from the following data

x	50	60	70	80	90
y	19.96	36.65	58.81	77.21	94.61

15. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's 1/3 rule, given

x	0	1	2	3	4	5	6
f(x)	1	0.5	0.2	0.1	0.0588	0.0385	0.027

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Set-02

Sixth Semester B.Sc. Degree Model Question Paper

(CBCS Scheme)

Mathematics

Paper 6.1 – Complex Analysis and Numerical Methods

Time: 3 Hours *Max.Marks:90*

Instruction to Candidates: Answer All the questions.

Part-A

I. Answer any SIX of the following. **(6 x 2 =12)**

1. Evaluate $\lim_{z\to(1+i)} \frac{z^2-z+1-i}{z^2-2z+2}$.
2. Show that $f(z) = xy + iy$ is not an analytical function.
3. Define singularity of a complex function. Give an example.
4. Evaluate $\int_c (\bar{z})^2 dz$ around the circle $|z| = 1$.
5. State Morera's theorem.
6. Show that $y_3 = y_0 + 3\Delta y_0 + 3\Delta^2 y_0 + \Delta^3 y_0$.
7. Evaluate $\int_0^6 y dx$ by Trapezoidal Rule from the following table:

X	0	1	2	3	4	5	6
Y	0.146	0.161	0.176	0.19	0.204	0.217	0.230

Part-B

II. Answer any SIX of the following. **(6 x 3 =18)**

1. Show that $|z-1|^2 + |z+1|^2 = 4$ represents a unit circle.
2. If $f(z)$ is analytic in an open set S and $f'(z) = 0$ for all $z \in S$, then show that $f(z)$ is a constant.
3. Prove that if $f(z) = u(x,y) + iv(x,y)$ is an analytic function, then u and v are harmonic.
4. Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the line $y = x$ and $y = x^2$.
5. Evaluate $f(a) = \int_c \frac{4z^2+z+5}{z-a} dz$ where C is the ellipse $9x^2 + 4y^2 = 36$ find $f(1), f(i)$.
6. Estimate the missing term of the following

x	0	1	2	3	4
F(x)	1	3	9	-	81

7. Estimate $f(7.5)$ from the table

x	5	6	7	8
F(x)	125	216	343	512

PART-C

III. Answer any FOUR of the following . **(4 x 5 = 20)**

1. Show that $arg(\frac{z-1}{z+1}) = \frac{\pi}{3}$ represents a circle and also find its centre and radius.
2. Prove that the necessary conditions for the function $f(z) = u + iv$ to be analytic is $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$.
3. Show that $f(z) = logz$ is analytic and hence prove that $f'(z) = \frac{1}{z}$.
4. Find the orthogonal trajectories of the family of the curves $e^{-x}cosy + xy = c$.
5. Find the radius of convergence of $\sum \frac{1}{n^n} z^n$.

IV. Answer any FOUR of the following . **(4 x 5 = 20)**

6. Show that (a) $\int_c \frac{dz}{z-a} = 2\pi i$. (b) $\int_c \frac{dz}{(z-a)^n} = 0$, $n=2,3,4.....$ where $C: |z-a| = r$, r>0.
7. State and prove Cauchy's Integral theorem.
8. Evaluate $\int_c \frac{z}{(z^2+1)(z^2-9)} dz$. where C: |z|=2.
9. Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where C is the circle |z|=1.5.
10. State and prove Cauchy's Inequality.

V. Answer any FOUR of the following . **(4 x 5 = 20)**

11. If $f(x)$ is a polynomial of nth degree in x, then prove that $\Delta^{n+1} f(x) = 0$.
12. By using Lagrange's Interpolation formula for the value of $f(x)$ for $x = 0$ from the given table.

x	-1	-2	2	4
F(x)	-1	-9	11	69

13. By means of Newtons forward interpolation formula ,find the value of $f(4.5)$ from the table:

x	4	6	8	10	12	14
F(x)	48	100	294	900	1210	2028

14. Find $y'(1.6)$ and $y''(1.6)$ from the following table:

X	1	1.1	1.2	1.3	1.4	1.5	1.6
F(x)	7.989	8.403	8.781	8.129	9.451	9.750	10.031

15. Evaluate $\int_0^1 \frac{x}{1+x^2} dx$ by using Simpson's 3/8 rule dividing into 3 equal parts .Hence find the approximate value of $\log\sqrt{2}$.

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Set-03

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Mathematics

Paper 6.1 – Complex Analysis and Numerical Methods

Time: 3 Hours Max. Marks : 90

Instruction to Candidates: Answer All the questions.

Part-A

I. Answer any SIX of the following. (6 x 2 =12)

1. Find the real and imaginary parts of the complex function $f(z) = \log(z)$.
2. Show that the function $f(z) = \bar{z}$ is not differentiable anywhere in the complex plane.
3. Define absolute convergence and circle of convergence of a power series.
4. Evaluate $\int_0^{1-i} z\ dz$ along the line $y = -x$.
5. Find the singularities of the function $f(z) = \frac{1}{z^2-4z+5}$.
6. Obtain the value of $\Delta f(x)$ where $f(x) = x(x-3)$.
7. Write the Lagrange's formula to obtain the interpolating polynomial for the set of points $(x_0, y_0), (x_1, y_1), (x_2, y_2)$ and (x_3, y_3).

Part-B

II. Answer any SIX of the following . (6 x 3 = 18)

1. Prove the $\lim_{z\to 0} \frac{\bar{z}}{z}$ does not exist.
2. Obtain the harmonic conjugate of $u = 2x(1-y)$ using C-R equations.
3. Obtain the power series expansion of the function $f(z) = \frac{1}{z-2}$ about $z = 0$.
4. If $f(z)$ is analytic in a region R, then prove that $\int_a^b f(z)dz$ is independent of the path in R joining any two points a and b in R.
5. State the fundamental theorem of algebra. Verify the same for $z^2 + 2z + 5 = 0$.
6. Construct the difference table for the polynomial $f(x) = x^4 - 3x^3 + 2x^2 - 5$ taking $x_0 = 0$ and $x_n = 4$.
7. Using trapezoidal rule, find $\int_0^1 f(x)dx$ from the following table:

x	0	0.25	0.5	0.75	1
$f(x)$	1	4	8	10	13

Part-C

III Answer any FOUR of the following. **(4 x 5 = 20)**

1. Express $f(z) = \sinh z$ in terms of its real and imaginary parts. Hence verify that it is analytic everywhere in the complex plane. Further, show that $f'(z) = \cosh z$.
2. Prove that $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$ are the necessary conditions for $f(z) = u(x,y) + iv(x,y)$ to be analytic.
3. Prove that if $f(z) = u(r,\theta) + iv(r,\theta)$ is analytic, then u and v satisfy the Laplace equation $\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{1}{r}\frac{\partial^2 \varphi}{\partial \theta^2} = 0$.
4. Show that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Further, obtain its harmonic conjugate. Also, find a function $f(z)$ whose real part is u.
5. Show that the series $\sum_{n=1}^{\infty} \frac{z^n}{n(n+1)}$ converges absolutely for $|z| \leq 1$.

IV. Answer any FOUR of the following . **(4 x 5 = 20)**

6. Evaluate $\int_C |z|^2 dz$ where C is made up of the straight line from (0,0) to (1,0) and then the straight line from (1,0) to (1,1).
7. State and prove Cauchy's integral theorem.
8. Evaluate $\int_C \frac{\cos \pi z}{z^2 - 1} dz$, where C is the circle $|z| = 2$ using Cauchy's integral formula.
9. State and prove Cauchy's inequality.
10. Prove that every polynomial equation $P(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n = 0$ with $n \geq 1$ and $a_n \neq 0$ has exactly n roots.

V. Answer any FOUR of the following . **(4 x 5 = 20)**

11. With usual notations, prove that $E = 1 + \Delta$ and $\nabla = 1 - E^{-1}$.
12. Using interpolation, obtain a cubic polynomial which takes the following values:

x	0	1	2	3
$f(x)$	1	2	1	10

13. Find the approximate value of $f(6)$ from the following table using interpolation:

x	1	2	3	7
$f(x)$	2	4	8	128

14. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 54$ from the following table:

x	50	51	52	53	54
$f(x)$	3.68	3.7	3.73	3.76	3.78

15. Using Weddle's rule, obtain the approximate value of $\int_0^3 \frac{1}{1+x} dx$, correct to three decimal places, choosing $h = 0.5$.

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