

TUMKUR UNIVERSITY

DEPARTMENT OF STUDIES & RESEARCH IN MATHEMATICS

Syllabus for Ph. D. Entrance Test (2022 -23)

PART A: Research Methodology

1. Teaching: Nature, objectives, characteristics and basic requirements, Learner's characteristics, Factors affecting teaching, Methods of teaching, Teaching aids, Evaluation systems.
2. Research Aptitude: Research Meaning, Characteristics and types; Steps of research; Methods of research; Research Ethics; Paper, article, workshop, seminar, conference and symposium.
3. Reviewing the literature.
4. Defining the Research Problem.
5. Research Design: Meaning of Research Design, Need for Research Design, Features of a Good Design, Important Concepts Relating to Research Design.
6. Reasoning (Including Mathematical), Logical Reasoning.
7. Interpretation and Report Writing, Thesis writing, it's characteristics and format.
8. Information and Communication Technology (ICT).

PART B: Cognate/Core subject

1. Algebra & linear algebra

Permutations, Combinations, Pigeon-hole principle, Inclusion-exclusion principle, Derrangements.

Fundamental theorem of arithmetic, Divisibility in \mathbb{Z} , Congruences, Chinese remainder theorem, Euler's ϕ -function, Primitive roots.

Groups, Subgroups, Normal subgroups, Quotient groups, Homomorphisms, Cyclic groups, Permutation groups, Cayley's theorem, Class equations, Sylow theorems.

Rings, Ideals, Prime and Maximal ideals, Quotient rings, Unique factorization domain, Principal ideal domain, Euclidean domain, Polynomial rings and Irreducibility criteria.

Fields, Finite fields, Field extensions, Galois theory.

Vector space, Subspace, Linear dependence, Basis, Dimension, Algebra of linear transformations, Algebra of matrices, Rank and determinant of matrices, Linear equations, Eigenvalues and Eigenvectors, Cayley-Hamilton theorem, Matrix representation of linear transformations, Change of basis, Canonical forms, Diagonal forms, Triangular forms, Jordan forms, Inner product spaces, Orthonormal basis, Quadratic forms, Reduction and classification of quadratic forms.

2. Real & Complex Analysis

Elementary set theory, Finite, Countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, Supremum, Infimum.

Sequences and series, Convergence, \limsup , \liminf . Bolzano Weierstrass theorem, Heine-Borel theorem, Continuity, Uniform continuity, Differentiability, Mean value theorem, Sequence and series of functions, Uniform convergence, Riemann sums and Riemann integral, Improper integrals, Monotonic functions, Types of discontinuity, Functions of bounded variation, Lebesgue measure, Lebesgue integrals. Functions of several variables, Directional derivatives, Partial derivative, Derivative as a linear transformations, Inverse and implicit function theorems.

Algebra of complex numbers, The Complex plane, Polynomials, Power series, Transcendental functions such as exponential, Trigonometric and Hyperbolic functions. Analytic functions, Cauchy-Riemann equations, Contour integrals, Cauchy theorems, Cauchy integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem, Taylor's series, Laurent series, Calculus of residues, conformal mapping, Mobius transformations.

3. Topology & Functional Analysis

Topology – Basis, Dense sets, Subspace and product topology, Separation axioms, Connectedness and Compactness.

Metric spaces, Compactness, Connectedness, Normed linear spaces, Spaces of continuous functions, Banach spaces, Hilbert spaces.

4. Differential Equations

Ordinary differential equations-Existence and Uniqueness of solutions of initial value problems for first order ordinary differential, Singular solutions of first order ODEs, system of first order ODEs.

General theory of homogeneous linear ODEs, Variation of parameters, Sturm-Liouville boundary value problem.

Partial differential equations- Lagrange and Charpit's method for solving first order PDEs, Cauchy problem for first order PDEs.

Classification of second order PDEs, General solution of higher orders with constant coefficients, Method of separation of variables for Laplace, Heat and wave equations.

Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema, Variational methods for BVP in ODE and PDE.

5. Numerical Analysis and Mathematical Methods

Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using

Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange's, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picards, Euler, modified Euler and Runge-Kutta method. Linear integral equations of first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and Eigen functions, Resolvent kernel, Integral transforms- Laplace transform, Fourier transform , Z-transform, Perturbation methods. Calculus of Variation.

6. Classical, Continuum and Fluid Mechanics

Generalized coordinates, Lagrange's equations, Hamilton's canonical equations, Hamilton's principle and principle of least action, Two-dimensional motion of rigid bodies, Euler's dynamical equations for the motion of a rigid body about an axis, Theory of small oscillations.

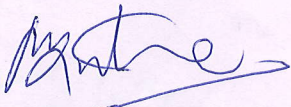
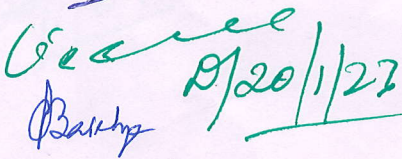

Co-ordinate transformations, Calculus of tensor, Continuum hypothesis, Material derivative, Stress tensor.

Law of conservation mass, momentum and energy, Motion of inviscous and viscous fluid, Dimensional analysis, Boundary layer theory.

7. Graph theory

Introduction to graph theory, Eulerian and Hamiltonian graphs, Covering and independence number (vertex and edge version), Connectivity, Menger's theorem, Planarity, Euler Identity, Non planar graphs, Maximal planar graph Outer planar graphs, Characterization of planar graphs, Geometric dual, Crossing number, Colorings, Bichromatic graphs, Colorings in critical graphs, Relation between chromatic number, Directed graphs, Tournament, Cyclic and transitive tournament, Spanning path in a tournament, Tournament with a Hamiltonian path, Strongly connected tournaments.

Signature of departmental council members

Name	Signature
1) Dr. Patil Mallikarjun B.	
2) Registrar (Evaluation)	
3) Dr. Chandrali Bhaishya	
4) Dr. Boregowda H. S	